Unit 1: Number 1 - Red indicates higher tier only -

Shaw Education Trust | The Orme Academy

Knowledge Organiser - Mathematics

Key Words

- *Factor*: Integers that multiply together to get another number
- *HCF*: Highest common factor of two or more numbers
- *Indices:* How many times to use a number in a multiplication
- *Integer*: A whole number
- *LCM*: Lowest common multiple of two or more numbers
- *Multiple*: Found by multiplying any number by a positive integer
- *Operation*: A mathematical process (+, -, x, ÷)
- *Prime Number*: A number with exactly two factors, 1 and itself

Adding/Subtracting Fractions

$\frac{3}{4} - \frac{1}{12}$

12 is a multiple of 4 (4 x 3) so, using equivalent fractions we can say $\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12} - \frac{1}{12} = \frac{8}{12}$

$= \frac{2}{3}$

$\frac{1}{2} + \frac{2}{3} + \frac{1}{6}$

Here, we know that 2 and 3 share a common multiple of 6, so we can say $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$

$\frac{3}{6} + \frac{4}{6} + \frac{1}{6} = \frac{8}{6}$

Simplify

$= \frac{4}{3}$

$= 1\frac{1}{3}$

Dividing Fractions

$\frac{2}{3} \div \frac{5}{7} \quad \frac{2}{3} \times \frac{7}{5} \quad \frac{2 \times 7}{3 \times 5} \quad \frac{14}{15}$

Inequalities

> greater than

< less than

≥ greater than or equal to

≤ less than or equal to

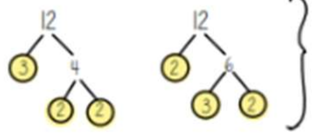
= equal to

≠ not equal to

$5 > 3$	5 is greater than 3
$2 + 2 = 4$	2+2 is equal to 4
$5 - 3 \times 2 \neq 4$	5-3x2 is not equal to 4
$x \leq 3$	x is less than or equal to 3

Product of Prime Factors

Write 12 as a product of it's prime factors

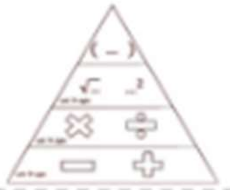


Both of these trees represent the same decomposition

$12 = 2 \times 2 \times 3$

$= 2^2 \times 3$

Order of Operations



$4 - 8 \times 2 + 12 \div 4$

So first we do the multiplication/division left to right; $4 - 16 + 3$

Now we do the addition/subtraction from left to right $-12 + 3 = -9$

Multiplying Fractions

$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

$\frac{5}{7} \times \frac{14}{15} = \frac{5 \times 14}{7 \times 15} = \frac{70}{105}$

$= \frac{2}{3}$

$1\frac{1}{2} \times 2\frac{1}{3}$

$\frac{3}{2} \times \frac{7}{3} = \frac{21}{6} = \frac{7}{2}$

$= 3\frac{1}{2}$

Dividing Decimals

The place holder is very important in division

All of these give the same solution

$1.5 \div 0.05 \rightarrow 15 \div 0.5 \rightarrow 150 \div 5$

Multiply both values until the divisor becomes an integer

Method 1

$0.12 \div 0.003$

$1.2 \div 0.03$

$12 \div 0.3$

$120 \div 3$

$= 40$

Method 2

$0.12 \div 0.003$ becomes $\frac{0.12}{0.003}$

Which we can rewrite as

$\frac{120}{3} = 40$

Prime Numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

LCMs

What is the LCM of 6 and 8?

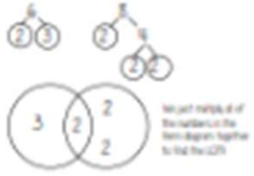
6 - 6, 12, 18, 24, 30

8 - 8, 16, 24, 32, 40

The first time their multiples match is 24 therefore

the LCM of 6 and 8 is 24

What is the LCM of 6 and 8?



LCM of 6 and 8 = 3 x 2 x 2 x 2

= 24

HCFs

What is the HCF of 6 and 8?

6 - 1, 2, 3, 6

8 - 1, 2, 4, 8

The biggest number which is a factor of both 6 and 8 is 2, therefore

the HCF of 6 and 8 is 2

What is the HCF of 6 and 8?

HCF of 6 and 8 = 2

Product Rule for Counting

To find the total number of outcomes for two or more events, multiply the number of outcomes for each event together.

Unit 2: Algebraic Manipulation - Red indicates higher tier only - Knowledge Organiser - Mathematics

Key Words

- *Binomials* – expressions with two terms (e.g., x + 3)
- *Coefficients* – numbers in front of variables (e.g., the 2 in 2x)
- *Equations* – expressions with equal signs (e.g., x + 3 = 7)
- *Expanding* – to multiply out of brackets
- *Expressions* – mathematical statements
- *Factorising* – to divide expressions into brackets
- *Formulae* – a mathematical rule that uses letters to represent amounts which can be changed (e.g., *b* x *h* for area of a rectangle)
- *Identities* – equations which are always true (e.g., 2 x y = 2y)
- *Inequalities* – expressing terms as unequal to one another, using >, <, ≥, ≤, or ≠
- *Quadratics* – expressions with a square terms as the highest power (e.g., $x^2 - 3x + 1$)
- *Surds* – an expression that includes a square, cube, or other root
- *Terms* – an individual component in an expression (e.g., the x in 2x + 5)
- *Variables* – values which can change, expressed as letters

Collecting Like Terms

$3a$ and $+2a$ are like terms

$+4b$ and $-2b$ are also like terms, but they are different to the terms with the letter a. The plus or minus sign in front of a term belongs to that term.

$$3a + 4b + 2a - 2b = 3a + 2a + 4b - 2b$$
$$= 5a + 2b$$

Expanding Single Brackets

6(x-3) = 6x-18	Use your negative rules
6x(x+5)= $6x^2$+30	Use index laws when multiplying powers
-2 (4+3x) = -8-6x	Take care with signs
y(y^2+9) -3y = y^3+6y	Simplify your answers after you have expanded.

Composite & Inverse Functions

If $h(x) = x^2$ and $f(x) = x - 5$ we can find an expression for $fh(x)$:

$fh(x) = f[h(x)]$

$= f[x^2]$ ← apply the function h first (squaring)

$= x^2 - 5$ ← then apply the function f (subtracting 5)

Example: Find the inverse of $f(x) = 5x + 3$

write the function using a "y" → $5y + 3 = x$ ← set equal to "x"

$5y = x - 3$

$y = \frac{x-3}{5}$ ← rearrange to make y the subject

use f^{-1} notation → $f^{-1} = \frac{x-3}{5}$

Algebraic Notation

ab = *a* x *b*

3*y* = *y* + *y* + *y* = 3 x *y*

a^2 = *a* x *a* & a^3 = *a* x *a* x *a*

a^2b = *a* x *a* x *b*

a/b = *a* ÷ *b*

Expanding Double Brackets

$(2x + 2)(x - 6)$

×	$2x$	$+2$
x	$2x^2$	$+2x$
-6	$-12x$	-12

$2x^2 + 2x - 12x - 12$

$2x^2 - 10x - 12$

Index Laws

$x^a \times x^b$	x^{a+b}
$x^a \div x^b$	x^{a-b}
x^{a^b}	x^{ab}

Function Machines

input → × 3 → output

input ← ÷ 3 ← output

Unit 3: Number 2 - Red indicates higher tier only - Knowledge Organiser - Mathematics

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Estimation

Estimate the value of 28 x 48

If we round both to 1 sf, this gives;
30 x 50 = 1500

Therefore $28 \times 48 \approx 1500$

Rounding to Significant Figures

Rounding to 1 significant figure (1 sf)

- Round 1394 to 1 sf = 1000
- Round 265 to 1 sf = 300
- Round 32 to 1 sf = 30
- Round 1.87 to 1 sf = 2
- Round 0.439 to 1 sf = 0.4
- Round 0.008722 to 1 sf = 0.009
- Round 0.0005043 to 1 sf = 0.0005

Rounding to 2 significant figures (2 sf)

- Round 1394 to 2 sf = 1400
- Round 265 to 2 sf = 270
- Round 32 to 2 sf = 32
- Round 1.87 to 2 sf = 1.9
- Round 0.439 to 2 sf = 0.44
- Round 0.008722 to 2 sf = 0.0087
- Round 0.0005043 to 2 sf = 0.00050

Converting to and from Standard Form

Converting ordinary numbers into standard form

$A \times 10^n$ — n: Any integer; A: Any number between 1 and 10

Examples

700	12500	0.00034
$= 7 \times 10 \times 10$	$= 1.25 \times 10 \times 10 \times 10 \times 10$ (must be between 1 and 10)	(Remember a negative power doesn't make the answer negative, just close to 0)
$= 7 \times 10^2$	$= 1.25 \times 10^4$	$= 3.4 \times 10^{-4}$

Converting standard form into ordinary numbers

Example 1	Example 2	Non-Examples
2×10^3	4.12×10^2	$12 \times 10^2 = 1200$ (must be between 1 and 10)
$= 2 \times 10 \times 10 \times 10$	$= 4.12 \times 10 \times 10$	$1.84 \times 10^{?} = 14.62$ (must be an integer!)
= 2000	= 412	$6.4 \times 8^{?} = 32768$ (must be a power of 10)

Simple Interest

I put £1000 in a bank account. It earns simple interest of 10% per year. How much will be in the account after 5 years?

INTEREST:
Simple interest means we calculate the interest the initial amount will earn and add that amount on each year

10% of £1000 = £100

So each year, the account will gain £100 interest.

£1000 + (£100 x 5) — 5 years
= £1500

Percentage of an Amount

300

100% of 300 = 300
10% of 300 = 30 (÷10)

Find 30% of 240

100% of 240 = 240
10% of 240 = 24 (÷10)
30% of 240 = 72 (x3)

A bar model to help visualise it:
24 x 3 = 72

(Finding 10% is always a good place to start!)

Find 81% of 480

100% of 480 = 480
10% of 480 = 48
1% of 480 = 4.8

100% of 480 = 480
10% of 480 = 48
80% of 480 = 384

(80% + 1% = 81% so we need to add 4.8 and 384)

81% of 480 = 388.8

Multipliers

What multiplier would represent an increase of 15%?

We are finding 100% + 15%, so 115%.

As a decimal this is 1.15

What multiplier would represent an decrease of 15%?

We are finding 100% - 15%, so 85%.

As a decimal this is 0.85

Percentage Increase/Decrease

12% increase means we have 112% of the original price. So we are now finding 112% of £400

100% of £400 = £400
10% of £400 = £40
2% of £400 = £8

112% of £400 = £448

Recurring Decimal to Fraction

Convert $0.\dot{1}\dot{2}$ to a fraction

$$\text{Let } x = 0.\dot{1}\dot{2}$$
$$100x = 12.\dot{1}\dot{2}$$
$$99x = 12 \quad (\div 99)$$
$$x = \frac{12}{99} = \frac{4}{33}$$

$$\begin{array}{r} 12.\dot{1}\dot{2} \\ -\ 0.\dot{1}\dot{2} \\ \hline 12.00 \end{array}$$

Standard Form Arithmetic

$(2.1 \times 10^6) + (3.3 \times 10^3)$

Foolproof method: convert both numbers to ordinary numbers and then add

$(2.1 \times 10^6) + (3.3 \times 10^3)$
$2,100,000 + 3300$
$= 2,103,300$
$= 2.103 \times 10^6$

(You should leave your answer in the form given in the question)

$(2.1 \times 10^6) \times (3.3 \times 10^3)$

In multiplication and division problems, you can multiply the A values and the look at the powers of 10

$2.1 \times 3.3 \times 10^6 \times 10^3$
$= 6.93 \times 10^6 \times 10^3$ (Remember $a^m \times a^n = a^{m+n}$)
$= 6.93 \times 10^9$

$(2.8 \times 10^8) \div (7 \times 10^5)$

$\frac{2.8 \times 10^8}{7 \times 10^5} = \frac{0.4 \times 10^8}{10^5} = 0.4 \times 10^3$ (Remember $a^m \div a^n = a^{m-n}$)

BUT:
0.4×10^3 is not in standard form, as A is not a number between 1 and 10. So, $0.4 \times 10^3 = 400$
$= 4 \times 10^2$

Error Intervals (and Calculating with Them)

A = 30 (to the nearest whole number) — Error Interval for A: $29.5 \le A < 30.5$
B = 11.5 (to the nearest 1 decimal place) — Error Interval for B: $11.45 \le B < 11.55$
C = 300 (to the nearest 1 significant figure) — Error Interval for C: $250 \le C < 350$

Calculate the maximum value of A + B
UB of A + UB of B: 30.5 + 11.55 = 42.05

Calculate the minimum value of A x C
LB of A x LB of C: 29.5 x 250 = 7375

Calculate the maximum value of C ÷ B
UB of C ÷ LB of B = 350 ÷ 11.45 = 30.7 (2dp)