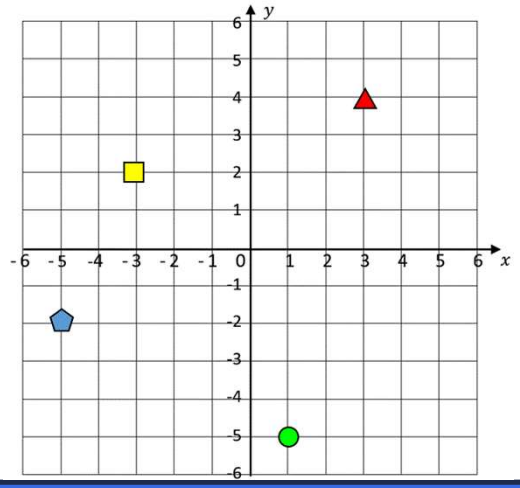
Unit 4: Linear Graphs - Red indicates higher tier only -

Knowledge Organiser - Mathematics

Coordinates in Four Quadrants



"Along the corridor, and up the stairs", meaning along the horizontal x-axis & up the vertical y-axis

Red Triangle = (3,4)
Yellow Square = (-3,2)
Green Circle = (1,-5)
Blue Pentagon = (-5,-2)

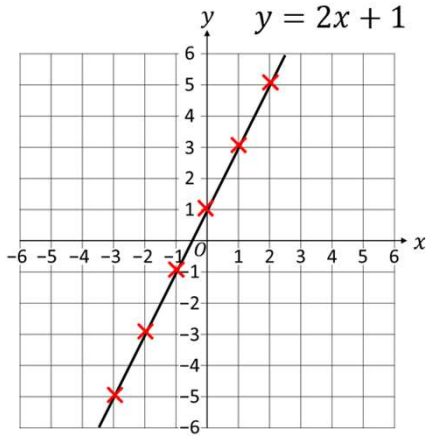
Drawing Linear (*Straight-Line*) Graphs

1) Complete a Table of Values.

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

2) Plot each pair of values as coordinates.

3) Join the points to make a line.



Parallel Lines

Example: *Find the equation of a line parallel to 2x + y = 4, that crosses the y-axis at 3.*

1. Rearrange the equation in the form y = mx + c
y = 2x + 4

2. *Identify the gradient*, in this case 2

Any line with the same gradient is parallel

3. *Write c as the y-intercept.* The line that has the same gradient and has a y-intercept of 3 is y = 2x - 3

Equation of a Straight Line

The general equation of any straight line is:

$$y = mx + c$$

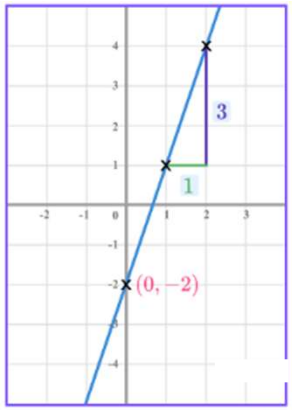
c is the y-**intercept** (where the line crosses the y-axis)

m is the **gradient** (steepness) of the line

Example The graph of the line $y = 3x - 2$

The gradient is 3

The y-intercept is -2, the coordinate $(0, -2)$



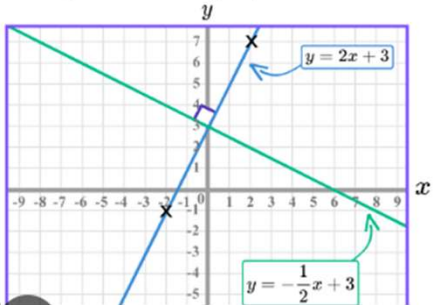
Gradient

The steepness of a line. Calculated as difference along the y-axis divided by difference along the x-axis, or dy/dx.

Perpendicular Lines

Perpendicular lines have gradients that multiply to give -1.

The gradients of perpendicular lines are the **negative reciprocals** of each other.



Example

The line $y = 2x + 3$ has a gradient of 2

The line $y = -\frac{1}{2}x + 3$ has a gradient of $-\frac{1}{2}$

$2 \times -\frac{1}{2} = \frac{-2}{2} = -1$

The gradients multiply to give -1

Unit 5: Solving Equations & Inequalities - Red indicates higher tier only - Knowledge Organiser - Mathematics

Solving One-Step Equations & Inequalities

To solve any equation or inequality we need to do the inverse of the operation that we see.

$t + 4 = 10$
$-4 \quad -4$
$t = 6$

The inverse of add is subtract and vice versa.

$c - 3 > 6$
$+3 \quad +3$
$c > 9$

$6y < 30$
$\div 6 \quad \div 6$
$y < 5$

The inverse of multiply is divide and vice versa.

$\frac{m}{7} = 4$
$\times 7 \quad \times 7$
$m = 28$

Solving Multi-Step Equations & Inequalities

To solve a two step equation or inequality we need to complete 2 inverse calculations in a specific order.

$6y + 2 = 32$
$-2 \quad -2$

Subtract first because the 2 is separate from the y.

$6y = 30$
$\div 6 \quad \div 6$

Divide because it is the inverse of multiplying.

$y = 5$

$\frac{w-5}{3} \geq 4$
$\times 3 \quad \times 3$

Multiply first because the entire expression is divided by 3.

$w - 5 \geq 12$
$+5 \quad +5$

Add because it is the inverse of subtracting.

$w \geq 17$

Solving Equations with Brackets

We must expand the bracket first and then solve by doing the inverse of the operations. We use the same method for inequalities.

$3(2x + 5) = 39$

Expand brackets first.

$6x + 15 = 39$
$-15 \quad -15$

The inverse of +15 is -15.

$6x = 24$
$\div 6 \quad \div 6$

The inverse of x 6 is ÷6.

$x = 4$

Solving Equations with Variables on Both Sides

To solve an equation or inequality with unknowns on both sides we need to collect all of the same terms together, still by looking at the inverse.

$5x - 20 \leq 3x + 4$
$-3x \quad -3x$

We subtract 3x from both sides because it is the smaller term of x.

$2x - 20 \leq 4$
$+20 \quad +20$

Then solve like a normal two step equation.

$2x \leq 24$
$\div 2 \quad \div 2$
$x \leq 12$

$2x - 10 = 5x + 2$
$-2x \quad -2x$

We subtract 2x from both sides because it is the smaller term of x.

$-10 = 3x + 2$
$-2 \quad -2$

Then solve like a normal two step equation.

$-12 = 3x$
$\div 3 \quad \div 3$
$-4 = x$

Solving Quadratic Equations by Factorising

Factorising quadratics is the opposite of expanding brackets and is used to solve quadratic equations. Most quadratic expressions require two brackets in the form $(x + p)(x + q)$

Example Factorise $x^2 + 2x - 15$

Factor pairs of 15 are 1 and 15, 3 and 5

We choose the factor pair 3 and 5 as these can be used to make 2 by addition or subtraction.

$x^2 + 2x - 15 = (x + 5)(x - 3)$

Solving Quadratics Using the Quadratic Formula

Quadratic equation in standard form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Simultaneous Equations

$2x = -3y + 16$
$5x - 4y = -6$

1: "Line up" the variables.

$2x + 3y = 16$
$5x - 4y = -6$

2: Determine which variable to eliminate. Make the coefficients opposites.

$2x + 3y = 16 \quad \rightarrow \quad 10x + 15y = 80$
$5x - 4y = -6 \quad \rightarrow \quad -10x + 8y = 12$

3: Add straight down. (One variable should be eliminated.)
4: Solve the resulting equation.

$10x + 15y = 80$
$+ \; -10x + 8y = 12$
$23y = 92$
$y = 4$

5: Substitute this result into either of the original equations.
6: Solve for the variable to find the ordered pair solution.

$2x + 3(4) = 16$
$x = 2$

7: Check the solution in both original equations.

Unit 6: Ratio & Proportion 1 - Knowledge Organiser - Mathematics

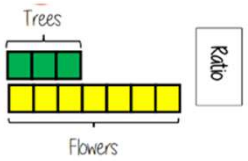
Key Words

- *Equivalent* – of equal value
- *Fraction* – represents how many parts per whole
- *Ratio* – a statement of how two (or more) numbers compare
- *Proportion* – a statement that links two ratios

Converting Between Ratios & Fractions

Trees: Flowers

3 : 7



Ratio

Fraction of trees = Number of parts of in group / Total number of parts

Flowers: 3 / 10

Fraction

Conversions Between Currencies

£1 = 90 Rupees

For every £1 I have 90 Rupees

£1 = 90 Rupees

× 10 / × 10

£10 = 900 Rupees

Convert 630 Rupees into Pounds

£1 = 90 Rupees

× 7 / × 7

630 ÷ 90 = 7

£7 = 630 Rupees

Ratios in the Form 1:*n* or *n*:1

This is asking you to simplify a ratio down until the part indicated represents 1.

Show the ratio 4:20 in the ratio of 1n

The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Best Buys

	4 pens costs £2.60	10 pens costs £6.00
'1 pen costs...'	£2.60 ÷ 4 = £0.65	£6.00 ÷ 10 = £0.60
'1-pound buys...'	4 ÷ 2.60 = 1.54 pens	10 ÷ 6 = 1.67 pens

Ratio & Scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life

1cm : 30cm

10cm : 300cm

× 10 / × 10

Dividing in a Ratio

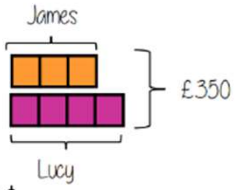
James and Lucy share £350 in the ratio 3:4

Work out how much each person earns

Model the Question

James : Lucy

3 : 4



Find the value of one part

Whole: £350

7 parts to share between (3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

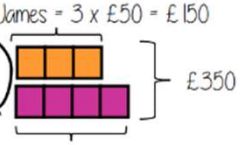
James : Lucy

× 50 3 : 4 × 50

£150 : £200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200



We can solve problems when the total is provided, but also if we know only one value, or if we know the difference between the two values:

B = 40

A : B

3 : 5

40 / 5 = 8

24 : 40

Difference = 40

A : B

3 : 5

40 / 2 = 20

60 : 100

Unit 6b: Ratio & Proportion 2 - Red indicates higher tier only - Knowledge Organiser - Mathematics

Key Words

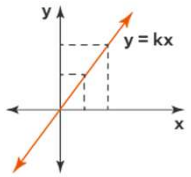
- *Compound Interest* – calculating interest on both the amount plus previous interest.
- *Chord* – a straight line joining the end of a curve.
- *Decay* – the process of reducing an amount by a consistent percentage over time.
- *Gradient* – the steepness of a line.
- *Growth* – where a value increases in proportion to its current value, e.g., doubling
- *Depreciation* – a decrease in the value of something over time.
- *Direct Proportion* – relationships wherein as one variable increases so does the other (e.g., cost of items & number of items).
- *Inverse Proportion* – relationships wherein as one variable increases the other decreases (e.g., number of workers & time taken to complete task).
- *Iteration* – the repetition of a mathematical procedure applied to the result of a previous application.
- *Proportion* – a statement that links two ratios
- *Tangent* – a straight line that meets a curve at a right-angle.

Compound Interest

Direct Proportion Equation & Graph

$y \propto x$

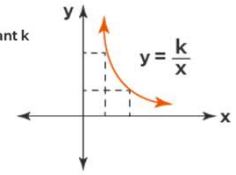
$y = kx$ for a constant k



Indirect Proportion Equation & Graph

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$ for a constant k



Iteration

The iterative formula to determine the next integer is $x_{n+1} = x_n + 1$ (next solution, current solution)

If we know that $x_0 = 3$ and we want to calculate the value of x_1, we substitute into the iterative formula above with $n = 0$: $x_1 = x_0 + 1$

$x_1 = 3 + 1 = 4$

We can use this process of **iteration** to find **approximate solutions** for equations.

Indirect Proportion

Example

4 people can take 3 hours to dig a trench.

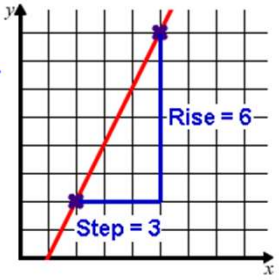
How long would it take 6 people working at the same rate?

	People	:	Time	
	4	:	3 hrs	
÷2				×2
	2	:	6 hrs	
×3				÷3
	6	:	2 hrs	

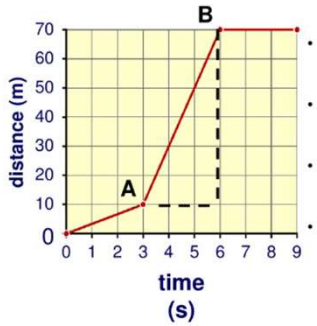
Gradient

Calculated as difference along the y-axis divided by

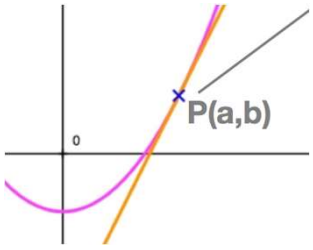
$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}} = \frac{6}{3} = 2$$



The gradient on a distance-time graph represents speed, since speed is the compound measure derived from distance divided by time.



- Gradient = Change in distance / Change in time
- Gradient = Distance at B – Distance at A / Time at B – Time at A
- Gradient = 70m – 10m / 6 – 3 = 60m / 3
- Gradient = 20m/s



The tangent to the curve at P has the same gradient as the curve at that point.