Unit 7: Perimeter, Area and Volume- Red title indicates higher tier

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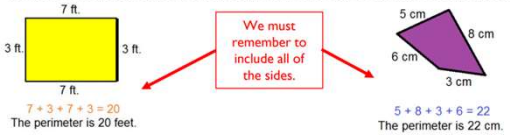
Knowledge Organiser - Mathematics

Perimeter and Area

Perimeter

Perimeter: This is the total distance around the outside of the shape.

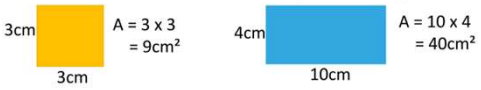


Area

Area: This is the space that a 2D shape takes up.

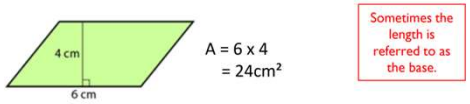
Squares and rectangles:

The formula is the same for both shapes: **A = Length x Width**



Parallelograms:

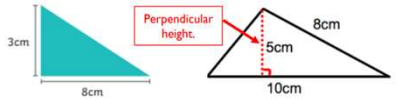
The formula is similar to a rectangle but instead of width we use the height. **A = Length x Height**



Triangles: To find the area of a triangle we use the following formula:

$$\text{Area} = \frac{\text{Base x perpendicular height}}{2}$$

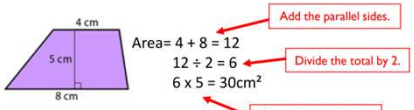
The formula is very similar to a rectangle but we must divide by 2 because a triangle is half the size of a rectangle.



Trapeziums: To find the area of a trapezium we use the following formula:

$$\text{Area} = \frac{(a+b)}{2} \times h$$

Where a and b are the parallel sides and h is the height.

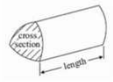


Volume and Surface area

The **volume** of an object is the amount of space that it occupies. It is measured in units cubed e.g. cm^3.

To calculate the volume of any prism we use:

$$\textit{area of cross section} \times \textit{length}$$



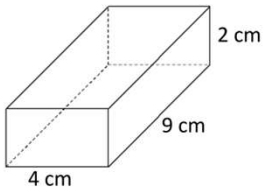
A **prism** is a 3D shape which has a continuous cross-section.

The **surface area** of an object is the sum of the area of all of its faces. It is measured in units squared e.g. cm^2.

Examples

$$\textbf{Volume} = 4 \times 9 \times 2$$

$$= \mathbf{72cm^3}$$



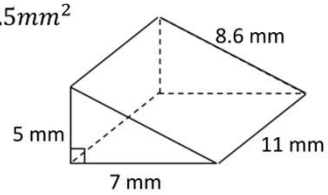
$$\textbf{Surface area:}$$
$$Front = 4 \times 2 = 8$$
$$Back = 4 \times 2 = 8$$
$$Side\ 1 = 9 \times 2 = 18$$
$$Side\ 2 = 9 \times 2 = 18$$
$$Bottom = 4 \times 9 = 36$$
$$Top = 4 \times 9 = 36$$
$$Total = \mathbf{124cm^2}$$

$$Area\ of\ triangle = \frac{5 \times 7}{2}$$

$$= 17.5mm^2$$

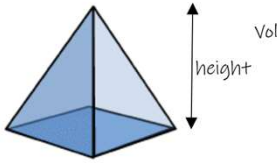
$$\textbf{Volume} = 17.5 \times 11$$

$$= \mathbf{192.5mm^3}$$



$$\textbf{Surface area:}$$
$$Front = \frac{7 \times 5}{2} = 17.5$$
$$Back = \frac{7 \times 5}{2} = 17.5$$
$$Side = 5 \times 11 = 55$$
$$Bottom = 7 \times 11 = 77$$
$$Top = 11 \times 8.6 = 94.6$$
$$Total = \mathbf{261.6cm^2}$$

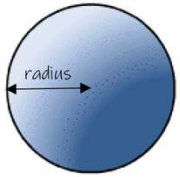
Volume of a Pyramid and Sphere



Volume = $\frac{1}{3}$ × area of base × perpendicular height

Surface area = area of base + area of all the triangles

Given to you in an exam!



Volume = $\frac{4}{3}$ x π x radius cubed

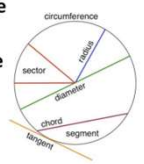
$= \frac{4}{3}\pi r^3$

Surface area = 4 x π x radius squared

$= 4\pi r^2$

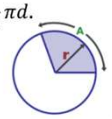
Circles

Parts of a circle



Circumference of a circle is calculated by πd and is the distance around the circle.

Arc length of a sector is calculated by $\frac{\theta}{360}\pi d$.



The **area** of a circle is calculated by πr^2

The **area of a sector** is calculated by $\frac{A}{360}\pi r^2$

Unit 8: Basic Trigonometry - Red title indicates higher tier only

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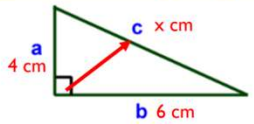
Knowledge Organiser - Mathematics

Pythagoras' theorem

You should always label the hypotenuse first.
This is the side facing the right angle.



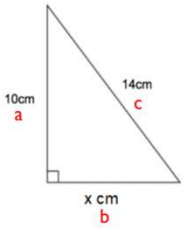
$a^2 + b^2 = c^2$

1) Substitute your values into the formulae:
$4^2 + 6^2 = x^2$
2) Work out the values that you can.
$16 + 36 = x^2$
$52 = x^2$
3) Now use inverse operations to isolate x.
$52 = x^2$
$(\surd)\ (\surd)$
$\sqrt{52} = x$
7.211102551 cm = x or 7.21 to 3 s.f

This is surd form. Sometimes you will be asked to leave your answer like this.

Pythagoras' Theorem – Shorter Sides

$a^2 + b^2 = c^2$



Sometimes you are asked to calculate the shorter sides, see below.
1) Substitute your values into the formulae:
$10^2 + x^2 = 14^2$
2) Work out the values that you can.
$100 + x^2 = 196$
3) Now use inverse operations to isolate x.
$100 + x^2 = 196$
$(-100)\quad (-100)$
$x^2 = 96$
$(\surd)\ (\surd)$
$\sqrt{96} = x$
x = 9.797958971 cm or 9.80cm to 3 s.f

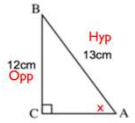
You need to get the numbers on one side, the x on it's own. An extra step is needed.

Trigonometric ratios

SOH	CAH	TOA
SINE = $\frac{OPP}{HYP}$	COSINE = $\frac{ADJ}{HYP}$	TANGENT = $\frac{OPP}{ADJ}$

Trigonometry – Calculating an angle

Trigonometry – Finding an angle
Calculate the size of angle BAC.



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverses to rearrange & isolate x.

$\sin(x) = \frac{12}{13}$

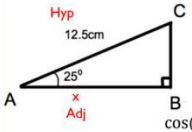
$x = \sin^{-1}\left(\frac{12}{13}\right)$

The inverse of sin, cos and tan are $\sin^{-1}$, $\tan^{-1}$, $\cos^{-1}$. They are found by pressing shift sin on your calculator.

$x = 67.38013505°$

I have not labelled the third side as it has no information on and I am not trying to calculate it.

Trigonometry – Calculating a length



Step 1 – Label the sides you need as O, A or H.
Step 2 – Use this to decide which trig ratio to use.
Step 3 – Substitute the given values into the formula.
Step 4 – Use inverse operations to rearrange & isolate x.

$\cos(25) = \frac{x}{12.5}$

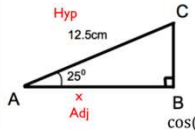
The inverse of dividing by 12.5 is multiplying.

$\cos(25) \times 12.5 = x$

$x = 11.32884734\ cm$

Don't round your answer, you get no marks for this!

Example 2



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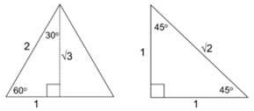
Exact values

For your exam you will need to learn the following values.
(Use the hand trick or triangles to help you learn them)

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–

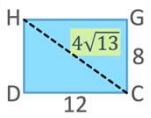


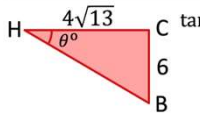
3D Pythagoras and Trigonometry

Ex2 ABCDEFGH is a cuboid. Work out the angle between the plane DCGH and the line HB.

Solution

$HC = \sqrt{12^2 + 8^2}$
$HC = 4\sqrt{13}$ cm

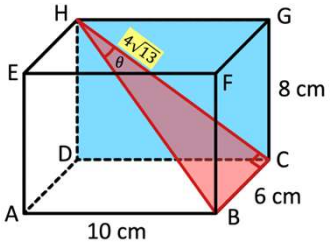




$\tan(\theta) = \frac{6}{4\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{6}{4\sqrt{13}}\right)$

$\theta = 22.6°$



Unit 9: Sequences - Red indicates higher tier only -

Knowledge Organiser - Mathematics

Key concepts

Arithmetic or linear sequences increase or decrease by a common amount each time.

Geometric series has a common multiple between each term.

Quadratic sequences include an n^2. It has a common second difference.

Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequences

-3 +3 +3 +3 +3

+1 4 7 10 13 16.....

a) State the nth term

$3n + 1$

Difference — The 0th term

b) What is the 100th term in the sequence?

$3n + 1$

$3 \times 100 + 1 = 301$

c) Is 100 in this sequence?

$3n + 1 = 100$

$3n = 99$

$n = 33$

Yes as 33 is an integer.

Geometric sequence e.g. ×3 ×3 ×3

4 12 36 108...

Quadratic sequence e.g. $n^2 + 4$ Find the first 3 numbers in the sequence

First term: $1^2 + 4 = 5$ Third term: $3^2 + 4 = 13$

Second term: $2^2 + 4 = 8$

Quadratic nth term

Nth term of a quadratic sequence

- Find the coefficient of n^2
- Multiply the value of n^2 for each term by this coefficient and subtract from the original sequence
- Find the nth term of the remaining linear sequence.

e.g. Find the nth term of the sequence 5, 7, 11, 17, 25, ...

Not a linear sequence as the 1st difference is not constant.

Not a geometric sequence as there is not a constant ratio (7 ÷ 5 = 1.4 but 11 ÷ 7 = 1.57...)

Sequence	5		7		11		17		25
1st difference		+2		+4		+6		+8	
2nd difference			+2		+2		+2		

As the second difference is 2, half of this gives us one lot of n^2

n	1	2	3	4	5
Sequence	5	7	11	17	25
n^2	1	4	9	16	25
Sequence minus n^2	4	3	2	1	0

The nth term of 4, 3, 2, 1, 0, ... is: $-n + 5$

This part is a linear sequence with a constant difference of -1 so we use a normal method for finding the nth term: see Position to Term Rules (nth term)

Therefore the nth term of the quadratic sequence is: $n^2 - n + 5$