Unit 10: Graphs 2 - Red indicates higher tier only - Knowledge Organiser - Mathematics

Shaw Education Trust · The Orme Academy

Key concepts

Quadratic function	An equation where the highest power of a variable (usually x) is 2, e.g. it contains an x^2 power but not an x^3 or higher. We use both the word function and equation to mean the same thing here.
Roots	The values of x in a quadratic equation which give a value of y = 0. On a graph, this is where it crosses the x-axis.

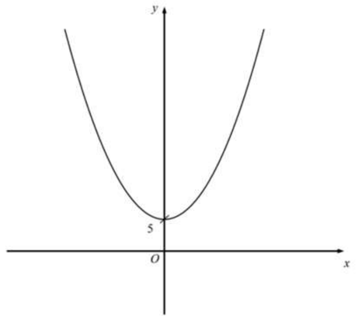
Sketching quadratics

All you need to know is whether it forms a u or a n shape, and identify where it would cross the y-axis.

e.g. sketch the graph $y = 3x^2 + 5$

a = 3 so is positive. So this is a u shape

c = 5, so crosses at (0, 5)



As it is a sketch, there is no need to plot any points accurately. The graph should be symmetrical about the y-axis and just label the crossing point.

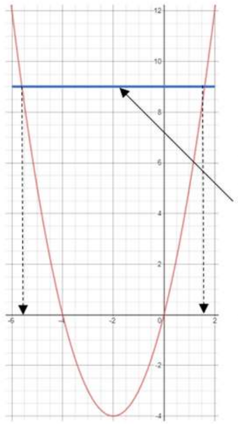
Plotting and using quadratic graphs

e.g. a) Complete the table of values for $y = x^2 + 4x$ and plot the graph

x	-6	-4	-2	0	2
y	12	0	-4	0	12

$y = (-6)^2 + 4 \times -6$
$y = 36 - 24 = 12$

As a quadratic graph is symmetrical, you will often see repeating values of y



b) Use the graph to find estimates for the solutions of $x^2 + 4x = 9$

We already have the graph of $y = x^2 + 4x$

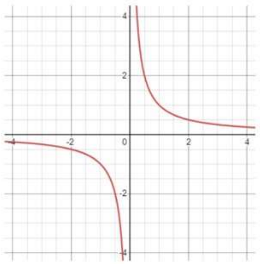
We draw on to the same axis the graph of $y = 9$

Where the 2 graphs intersect (cross) we read off the two x values.

So x = 1.5 and x = -5.5

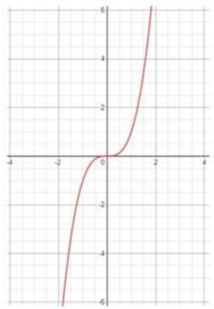
Reciprocal graphs

$y = \frac{1}{x}$



Cubic graphs:

$y = x^3$



Completing the square

Completing the Square (when $a = 1$)

A quadratic in the form $x^2 + bx + c$ can be written in the form $(x + p)^2 + q$

1. Write a set of brackets with x in and **half** the value of b.
2. Square the bracket.
3. Subtract $\left(\frac{b}{2}\right)^2$ and add c.
4. Simplify the expression.

You can **use the completing the square form** to help **find the maximum or minimum** of quadratic graph.

$y = x^2 - 6x + 2$

Answer:

$(x - 3)^2 - 3^2 + 2$

$= (x - 3)^2 - 7$

The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$

When $x = 3$, $y = 0 - 7 = -7$

Minimum point = $(3, -7)$

Completing the Square (when $a \neq 1$)

A quadratic in the form $ax^2 + bx + c$ can be written in the form $\mathbf{p}(x + q)^2 + r$

Use the same method as above, but factorise out a at the start.

Example

Complete the square of

$4x^2 + 8x - 3$

Answer:

$4[x^2 + 2x] - 3$

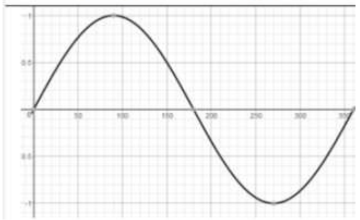
$= 4[(x + 1)^2 - 1^2] - 3$

$= 4(x + 1)^2 - 4 - 3$

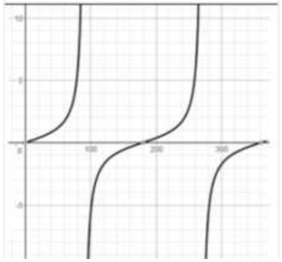
$= 4(x + 1)^2 - 7$

Trigonometric graphs

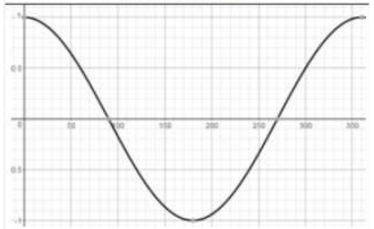
$y = \sin(x)$ for $0 \le x \le 360^o$



$y = \tan(x)$ for $0 \le x \le 360^o$



$y = \cos(x)$ for $0 \le x \le 360^o$



Unit 11: Solving inequalities - Red indicates higher tier only - Knowledge Organiser - Mathematics

Key concepts

Inequalities show the **range** of numbers that satisfy a rule.

$x < 2$ means x is less than 2

$x \leq 2$ means x is less than or equal to 2

$x > 2$ means x is greater than 2

$x \geq 2$ means x is greater than or equal to 2

On a **number line** we use circles to highlight the key values:

○ is used for less/greater than

● is used for less/greater than or equal to

Inequalities

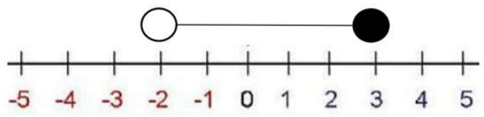
a) State the values of n that satisfy:

$$-2 < n \leq 3$$

Cannot be equal to 2 Can be equal to 3

-1, 0, 1, 2, 3

b) Show this inequality on a number line:



Solving Inequalities

Solve this inequality and represent your answer on a **number line:**

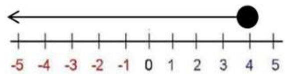
$5x - 6 \leq 14$

+6 +6

$5x \leq 20$

÷5 ÷5

$x \leq 4$



Solve this inequality and represent your answer on a **number line:**

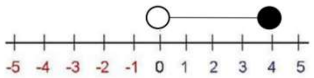
$4 < 3x + 1 \leq 13$

-1 -1

$3 < 3x \leq 12$

÷3 ÷3

$1 < x \leq 4$



Quadratic inequalities

Solving a quadratic inequality is very similar to solving a quadratic equation.

Step 1: Solve the equation to find the critical values.

Step 2: Sketch the curve

Step 3: Write down the appropriate inequality/inequalities

e.g. Solve $x^2 + 10x - 24 < 0$

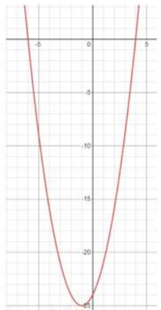
Start by solving: $x^2 + 2x - 24 = 0$ $(x + 6)(x - 4) = 0$ $x = -6$ and $x = 4$ (these are the critical values)

The curve is a positive quadratic so is a 'u' shaped parabola.

The roots of the equation are at x = -6 and x = 4, so this is where it crosses the x axis.

The curve is < 0 (below the x-axis) when it is between x = -6 and x = 4.

Therefore, the solution to $x^2 + 10x - 24 < 0$ is: $-6 < x < 4$



Note: if the question instead was,

Solve $x^2 + 10x - 24 > 0$ we now need the sections above the x-axis which are not connected and so the solution would have been $x < -6$ and $x > 4$

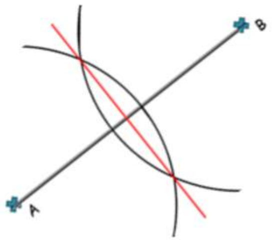
Unit 12: Angles and Constructions-

Red indicates higher tier

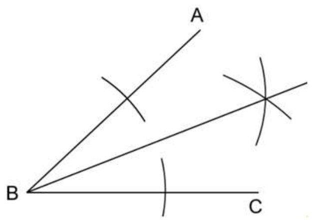
Knowledge Organiser - Mathematics

Constructions and Loci

Line Bisector



Angle Bisector



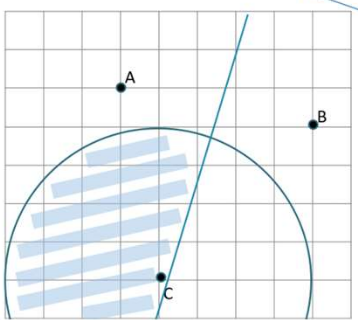
Loci

Shade the region that is:

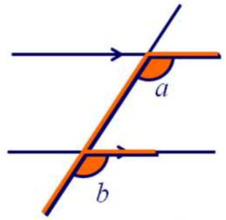
- closer to A than B
- less than 4 cm from C

Line bisector of A and B

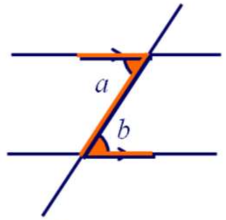
Circle with radius 4cm



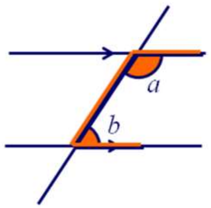
Angles in parallel lines



Corresponding angles are equal.

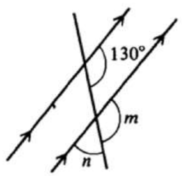


Alternate angles are equal.

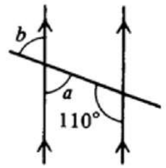


Co-interior angles add to 180°.

Example

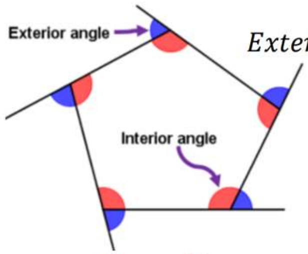


m = 130° as corresponding angles are equal.
n = 50° as angles on a line add to 180°

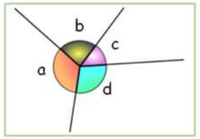


a = 70° as co-interior angles add to 180°
b = 70° as vertically opposite angles are equal

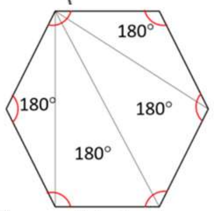
Angles in polygons



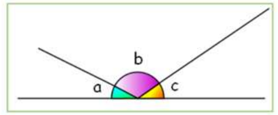
$$Exterior = \frac{360}{no.of\ sides}$$



Angles at a point add to 360°

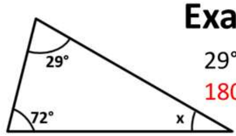


$$Sum\ of\ interior = 180° \times 4 = 720°$$

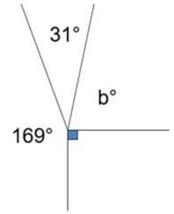


Angles on a line add to 180°

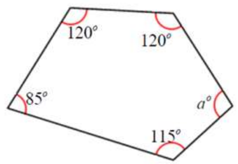
Examples



29° + 72° = 101°
180° – 101° = 79°



169° + 31°+ 90° = 290°
360° – 290° = 70°



120° + 120°+ 85° + 115° = 440°
540° – 440° = 100°