Unit 21: Trigonometry - Red indicates higher tier only -

Shaw Education Trust · The Orme Academy

Knowledge Organiser - Mathematics

Which Strategy Should I Use?

Is the triangle right-angled?

- Yes → Does the question involve any angles?
  - Yes → Use trig ratios SOHCAHTOA
  - No → Use Pythagoras' Theorem
- No → Do you know a side and an opposite angle?
  - Yes → Use the Sine rule
  - No → Use the Cosine rule

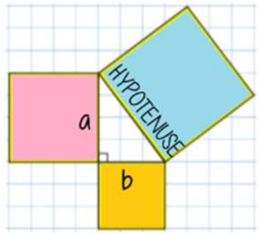
Exact Trigonometric Values

	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	*undefined*

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle.

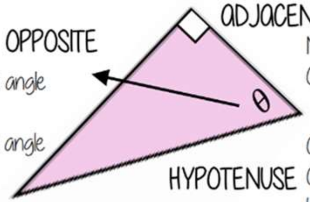
Pythagoras' Theorem

$Hypotenuse^2 = a^2 + b^2$



Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Labelling Right-Angled Triangles



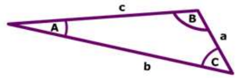
OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

The Sine Rule

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

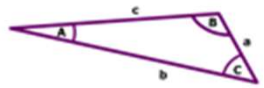
The Cosine Rule

This is the cosine rule to find a missing side:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

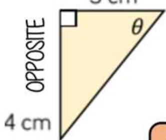
This is the rearranged cosine rule to find a missing angle:

$$Cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$



Using SOH CAH TOA to Find Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$Tan\theta = \frac{3}{4}$

$\theta = Tan^{-1}\frac{3}{4}$

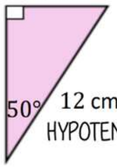
$\theta = 36.9°$

$\theta = Tan^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = Sin^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

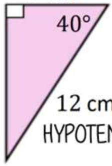
$\theta = Cos^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Using SOH CAH TOA to Find Sides



$Sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The $Sin(x)$ ratio is the same as the $Cos(90-x)$ ratio



$Cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

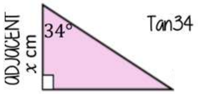
Substitute the values into the ratio formula

Equations might need rearranging to solve

$Tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula

$Tan34 = \frac{10}{x}$



Unit 22: Vectors

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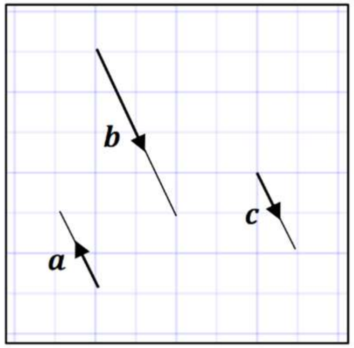
Key Words

- ***Column Vector*** – a matrix (coordinate with one number on top of another) of one column describing the movement from a point.
- ***Direction*** – the line or course something is travelling along.
- ***Magnitude*** – the length of a vector
- ***Parallel*** – straight lines that never meet
- ***Resultant*** – the vector that is the sum of two or more vectors.
- ***Scalar*** – a single number used to represent the multiplier when working with vectors.

Multiplying Vectors by a Scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

$\boldsymbol{a} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix}1\\-2\end{pmatrix}$

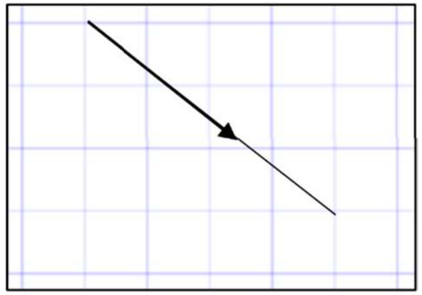
Vectors

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis →

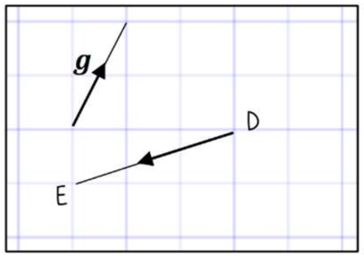
Movement along the y-axis →

$\begin{pmatrix}4\\-3\end{pmatrix}$ →



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E

$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$

Adding Vectors

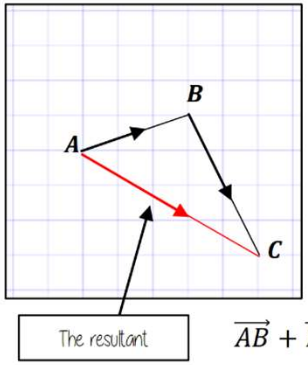
$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$

$\overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$

$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$

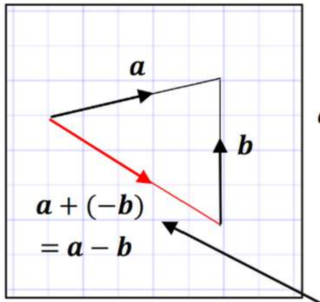
$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$



Look how this addition compares to the vector $\overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$

Addition & Subtraction of Vectors



$\boldsymbol{a} = \begin{pmatrix}5\\1\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}0\\4\end{pmatrix}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix}5+ & -0\\1+ & -4\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1